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*Solution by the PROPOSER.

The variation of $\int V dx$ may be divided into Δu and Δu_1 , the former arising supposing v constant, the latter from the variation of v . Thus $\Delta u = H + \int K \Delta y dx$, $\Delta u_1 = \int \frac{dV}{dv} \Delta v dx$. If the letters represent for V what their primes represent for V' , $\Delta u = \int (N' \Delta y + P' \Delta p + \dots) dx$. If $L = \frac{dV}{dv}$ and $I = \int L dx$,

$$\Delta u_1 = \int L \Delta v dx = I dv - \int I \frac{d \Delta v}{dx} dx = I (H' + \int K' \Delta y dx) - (H' + \int K' \Delta y dx)$$

(H' , K' denoting H' and K' when IN' , IP' , etc., are substituted for N' , P' , Then

$$\begin{aligned} \Delta u + \Delta u_1 &= \Delta y \left(P - \frac{dQ}{dx} \dots \right) + \Delta p \left(Q - \frac{dR}{dx} + \dots \right) + \dots \\ &+ \int \left(N - \frac{dP}{dx} + \frac{d^2 Q}{dx^2} - \dots \right) \Delta y dx + I \Delta y \left(P' - \frac{dQ'}{dx} + \frac{d^2 R'}{dx^2} \dots \right) \\ &+ I \Delta p \left(Q' - \frac{dR'}{dx} + \dots \right) + \dots + I \int \left(N' - \frac{dP'}{dx} + \frac{d^2 Q'}{dx^2} \dots \right) dx \\ &- \Delta y \left(IP' - \frac{dIQ'}{dx} + \dots \right) - \Delta p \left(IP' - \frac{dIQ'}{dx} + \dots \right) - \dots \\ &- \int \left(IN' - \frac{dIP'}{dx} + \frac{d^2 IQ'}{dx^2} \dots \right) dx. \end{aligned}$$

Also solved by G. B. M. Zerr.

205. Proposed by Z. T. JACKSON, St. Louis, Mo.

Evaluate† $\int_0^{\frac{1}{2}\pi} \log \sin x \, dx$.

Solution by J. E. SANDERS, Reinersville, Ohio.

$$u = \int_0^{\frac{1}{2}\pi} \log \sin x \, dx = \int_0^{\frac{1}{2}\pi} \log \sin(\tfrac{1}{2}\pi - x) \, dx = \int_0^{\frac{1}{2}\pi} \log \cos x \, dx.$$

$$\begin{aligned} \therefore 2u &= \int_0^{\frac{1}{2}\pi} (\log \sin x + \log \cos x) \, dx = \int_0^{\frac{1}{2}\pi} \log(\sin x \cos x) \, dx \\ &= \int_0^{\frac{1}{2}\pi} \log \frac{\sin 2x}{2} \, dx = \int_0^{\frac{1}{2}\pi} \log \sin 2x \, dx - \frac{\pi}{2} \log 2. \end{aligned}$$

*See Williamson's *Integral Calculus*, Sixth Edition, p. 275.

†Byerly, *Integral Calculus*, p. 102.

Let $2x = x'$, then

$$\int_0^{\frac{1}{2}\pi} \log \sin 2x \, dx = \frac{1}{2} \int_0^{\pi} \log \sin x' \, dx' = \int_0^{\frac{1}{2}\pi} \log \sin x \, dx.$$

$$\therefore 2u = u - \frac{\pi}{2} \log 2, \quad u = -\frac{\pi}{2} \log 2 = -\frac{\pi}{2} \log \frac{1}{2}.$$

Also solved by M. E. Graber, G. W. Greenwood, L. E. Newcomb, and G. B. M. Zerr.

206. Proposed by DR. O. E. GLENN, Drury College.

Evaluate $\int_0^1 (1-z^n)^m \frac{\partial}{\partial z} \log(1-z^n x^n) dz$, assuming $-1 < x^n < +1$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons. W. Va.

$$u = - \int_0^1 \frac{nx^n z^{n-1} (1-z^n)^m}{1-x^n z^n} dz. \quad \text{Let } 1-z^n = y, \text{ then we have}$$

$$u = - \int_0^1 \frac{x^n y^m}{1-x^n + x^n y} dy = - \int_0^1 \frac{y^m}{y+a} dy, \text{ where } \frac{1-x^n}{x^n} = a.$$

$$\begin{aligned} \therefore u &= - \int_0^1 \left(y^{m-1} - a y^{m-2} + a^2 y^{m-3} \dots (-1)^{m-1} a^{m-1} + \frac{(-1)^m a^m}{y+a} \right) dy \\ &= - \left[\frac{y^m}{m} - \frac{a y^{m-1}}{m-1} + \dots + (-1)^{m-1} a^{m-1} y + (-1)^m a^m \log(y+a) \right]_0^1 \\ &= - \left[\frac{1}{m} - \frac{a}{m-1} + \dots + (-1)^{m-1} a^{m-1} + (-1)^m a^m \log\left(\frac{1+a}{a}\right) \right] \\ &= - \left[\frac{1}{m} - \frac{(1-x^n)}{x^n(m-1)} + \frac{(1-x^n)^2}{x^{2n}(m-2)} + \dots + \frac{(-1)^{m-1} (1-x^n)^{m-1}}{x^{(m-1)n}} \right. \\ &\quad \left. + (-1)^{m+1} \frac{(1-x^n)^m}{x^{mn}} \log(1-x^n) \right]. \end{aligned}$$

DIOPHANTINE ANALYSIS.

128. Proposed by F. P. MATZ, Ph. D., Sc. D., Reading, Pa.

Required the highest powers of 2, 3, 5, 7, contained in (1000)!

I. Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

$$(1000)! = 2^{500} (500)! (1.3.5 \dots 999)$$

$$(500)! = 2^{250} (250)! (1.3.5 \dots 499)$$

.....

Proceeding thus we find the powers required are

$$2^{994}, \quad 3^{498}, \quad 5^{249}, \quad 7^{164}.$$